ON THE STRUCTURE OF ELECTROMAGNETIC FIELD GENERATED BY MOVING EXTERNAL CURRENT SOURCE IN A MAGNETIZED PLASMA

M. L. Khodachenko[*†], D. Langmayr[*‡], and H. O. Rucker[*‡]

Abstract

We present the first step of our theoretical analysis of the electromagnetic structure excited by a conductor, moving with the velocity V_0 in a hot magnetized plasma. The case of slow motion ($V_i << V_0 << V_e$) is considered within the frame of plasma kinetics. Of the primary interest for us are the fields, which are steady–state in the reference frame co–moving with the conductor. These fields are mainly due to the low–frequency quasi–stationary inductive electromagnetic mode. A characteristic frequency of the studied electromagnetic fields is $\omega = \mathbf{k} \cdot \mathbf{V}_0$. This means that the plasma dielectric properties appear to be in a strong dependence of the orientation of the wave vector $\mathbf{k}$ with respect to the velocity vector of the conductor, $\mathbf{V}_0$. As a first step of our investigation of the spatial structure of the electromagnetic environment created by a slowly moving conducting body in a magnetized plasma we replace the conductor by an elementary external non–self–consistent current source (thin line current). In this work we describe the mathematical methods, physical assumptions, and obtained solutions for this particular case. The presented problem has many applications for antenna and probe operations in plasmas as well as in space science, e. g., tethered satellites systems and the Io–Jupiter electromagnetic interaction. For our particular calculations in this paper we use as input parameters the specific circumstances in the Io torus. We demonstrate that the generated electromagnetic fields show a stretched configuration along the line of motion of the conductor, and have an oscillatory behavior with decay along the external magnetic field. Collision–less energy dissipation rates for the external line current moving in a magnetized plasma are calculated. They demonstrate a resonant behaviour with the maximum value reached for $V_0 \sim 0.7\ V_e$.

[*] *Space Research Institute, Austrian Academy of Sciences, Schmiedlstrasse 6, A–8042 Graz, Austria*

[†] *on leave from Institute of Applied Physics, Russian Academy of Sciences, Ulyanova str. 46, Nizhny Novgorod, Russia*

[‡] *Institute for Geophysics, Astrophysics, and Meteorology, University of Graz, Universitätsplatz 5, A–8010 Graz, Austria*

1 Introduction

This paper is devoted to the theoretical analysis of the electromagnetic interaction between an external current source moving with the velocity $\mathbf{V}_0$ and a hot magnetized plasma. This problem is usually considered within the frame of MHD–theory for applications in laboratory experiments, antennas, as well as in space physics [McKenzie, 1991]. Our interest in this rather general problem is inspired by some questions of Io–Jupiter physics. Indeed, the problem of the electromagnetic interaction between the satellite Io and the surrounding Jovian magnetospheric plasma can be formulated as a self–consistent problem of the interaction between a conducting body and a magnetized plasma in relative motion. The key point is that the specific plasma properties in the Io torus cause the situation that the collisions of plasma particles are not important up to several tens of Io radii, say up to $(60-80)\ R_{Io}$. Therefore, MHD approximation is inapplicable and the problem has to be treated within the frame of kinetic theory. Considering the Io–Jovian application of the stated problem implies that we appear to be in the situation of relatively slow motion of the conductor with respect to the Alfvén velocity, V_A, and thermal electron velocity, V_e, whereas the thermal ion velocity, V_i, remains to be less than the velocity of the conductor ($V_e \gg V_0 \gg V_i$).

The slow motion of an external current source means that in this situation the effects of plasma spatial dispersion, connected with the particle thermal motion, together with the anisotropy of the plasma dielectric properties due to the external magnetic field lead to a modification of the Alfvénic and magnetoacoustic wing–like structure of the generated fields near the moving current source [McKenzie, 1991], and produce some specific steady–state electromagnetic environment, formed by the non–propagating inductive electromagnetic fields [Gubchenko, 1989; Khodachenko and Gubchenko, 1990, 1997; Barnett and Olbert, 1986] co–moving with the source. These fields decay in space due to a collisionless energy dissipation and can be considered as a kind of a local magnetosphere of the moving current source [Barnett and Olbert, 1986; Khodachenko and Gubchenko, 1997]. Along with the influence the energy losses of the moving external current [Gubchenko, 1989; Khodachenko and Gubchenko, 1990; Khodachenko et al., 1998], inductive fields can be responsible for the appearance of electromagnetic structures near the source, where resonant charged particles will be effectively accelerated, and in their turn would manifest in the radiation features of the source.

The task of obtaining a self–consistent solution describing the structure of the electromagnetic fields and plasma in the vicinity of the conductor can be formulated in the form of an integral equation [Barnett and Olbert, 1986; Khodachenko et al., 1998]. The numerical analysis of this equation requires information about the plasma dielectric properties in terms of the dielectric tensor. The consideration of the non self–consistent problem, i.e., the investigation of the excited electromagnetic fields due to a given external elementary current source, provides solutions serving as tests for the general problem. The main aim of the present paper is to calculate the spatial structure of the excited electromagnetic environment assuming the external current source to be an infinite thin line current.

We are interested in the fields which are comoving with the conducting body. The characteristic frequency of such fields being steady–state in the reference frame of the conductor is $\omega = \mathbf{k}\cdot\mathbf{V}_0$. Thus, the frequency range of the generated fields is determined by the direction of the wave vector $\mathbf{k}$ with respect to $\mathbf{V}_0$, as well as by their total strengths [Khodachenko et al., 1998].

2 Basic equations

In the linear case, the electric field generated by any external current in a plasma is determined by the following wave equation

$$\hat{\Lambda}\tilde{\mathbf{E}}(\mathbf{k},\omega) = \frac{4\pi\omega i}{c^2}\tilde{\mathbf{j}}_s(\mathbf{k},\omega). \tag{1}$$

Here $\tilde{\mathbf{E}}(\mathbf{k},\omega)$ and $\tilde{\mathbf{j}}_s(\mathbf{k},\omega)$

$$\tilde{\mathbf{E}}(\mathbf{k},\omega) = \frac{1}{(2\pi)3}\int dt \int d3re^{i\omega t - i\mathbf{kr}}\mathbf{E}(\mathbf{r},t), \tag{2}$$

$$\tilde{\mathbf{j}}_s(\mathbf{k},\omega) = \frac{1}{(2\pi)3}\int dt \int d3re^{i\omega t - i\mathbf{kr}}\mathbf{j}_s(\mathbf{r},t), \tag{3}$$

correspond to the Fourier images of the electric field $\mathbf{E}(\mathbf{r},t)$ and the current source $\mathbf{j}_s(\mathbf{r},t)$, respectively. The tensor $\hat{\Lambda}(\mathbf{k},\omega)$ is connected with the dielectric tensor $\varepsilon_{ij}(\mathbf{k},\omega)$, and characterizes the plasma dispersion properties

$$\hat{\Lambda} = k^2\delta_{ij} - k_ik_j - \frac{\omega^2}{c^2}\varepsilon_{ij}(\mathbf{k},\omega). \tag{4}$$

The corresponding magnetic field can be calculated via the electric field

$$\tilde{\mathbf{B}}(\mathbf{k},\omega) = \frac{c}{\omega}[\mathbf{k}\times\tilde{\mathbf{E}}(\mathbf{k},\omega)]. \tag{5}$$

The specific circumstances of the Io–Jupiter interaction allow us to introduce some basic assumptions, i.e., the direction of the conductor (external current source) motion is perpendicular to the external magnetic field. Thus, we assume $\mathbf{V}_0$ to be directed along the x–axis and the external magnetic field $\mathbf{B}_0$ along z. As a next step, we take the external thin line current source only having an y–component non–zero. The Fourier image of such a current source is of the following form [Khodachenko et al., 1998]

$$\tilde{j}_{sy}(k_x,k_y,\omega) = -\frac{I_0}{(2\pi)^2}\delta(\omega - k_xV_0)\delta(k_y). \tag{6}$$

Due to the two δ–functions, the two integrations over ω and k_y can be trivially done and we obtain the expressions, which describe the respective components of the comoving electromagnetic fields

$$E_x(x,z) = \frac{16\pi V_0 I_0}{c^2(2\pi)^2}\left\{\int_0^\infty\int_0^\infty \sin(k_xx)\sin(k_zz)k_x^2k_zf_1(k_x,k_z)\,dk_xdk_z\right.$$

$$- \int_0^\infty \int_0^\infty \cos(k_x x)\sin(k_z z) k_x^2 k_z f_2(k_x,k_z)\ dk_x dk_z \Bigg\}, \tag{7}$$

$$E_y(x,z) = \frac{16\pi V_0 I_0}{c^2(2\pi)^2}\Bigg\{ \int_0^\infty \int_0^\infty \sin(k_x x)\cos(k_z z)\cdot k_x f_3(k_x,k_z)\ dk_x dk_z$$

$$+ \int_0^\infty \int_0^\infty \cos(k_x x)\cos(k_z z) k_x f_4(k_x,k_z)\ dk_x dk_z \Bigg\}, \tag{8}$$

$$E_z(x,z) = -\frac{16\pi V_0 I_0}{c^2(2\pi)^2}\Bigg\{ \int_0^\infty \int_0^\infty \sin(k_x x)\cos(k_z z) k_x \lambda_{xx} f_2(k_x,k_z)\ dk_x dk_z$$

$$+ \int_0^\infty \int_0^\infty \cos(k_x x)\cos(k_z z) k_x \lambda_{xx} f_1(k_x,k_z)\ dk_x dk_z \Bigg\}, \tag{9}$$

$$B_x(x,z) = -\frac{16\pi I_0}{c(2\pi)^2}\Bigg\{ \int_0^\infty \int_0^\infty \cos(k_x x)\sin(k_z z) k_z f_3(k_x,k_z) dk_x dk_z$$

$$- \int_0^\infty \int_0^\infty \sin(k_x x)\sin(k_z z) k_z f_4(k_x,k_z)\ dk_x dk_z \Bigg\}, \tag{10}$$

$$B_y(x,z) = \frac{16\pi I_0}{c(2\pi)^2}\Bigg\{ \int_0^\infty \int_0^\infty \sin(k_x x)\cos(k_z z)[k_x\lambda_{xx} - k_x k_z^2] f_2(k_x,k_z)\ dk_x dk_z$$

$$+ \int_0^\infty \int_0^\infty \cos(k_x x)\cos(k_z z)[k_x\lambda_{xx} - k_x k_z^2] f_1(k_x,k_z)\ dk_x dk_z \Bigg\}, \tag{11}$$

$$B_z(x,z) = \frac{16\pi I_0}{c(2\pi)^2}\Bigg\{ \int_0^\infty \int_0^\infty \sin(k_x x)\cos(k_z z) k_x f_3(k_x,k_z)\ dk_x dk_z \tag{12}$$

$$+ \int_0^\infty \int_0^\infty \cos(k_x x)\cos(k_z z) k_x f_4(k_x,k_z)\ dk_x dk_z \Bigg\}, \tag{13}$$

where

$$f_1(k_x,k_z) = \frac{\mathrm{Im}[\lambda_{zy}]\mathrm{Re}[\det\hat{\lambda}] - \mathrm{Re}[\lambda_{zy}]\mathrm{Im}[\det\hat{\lambda}]}{\mathrm{Re}^2[\det\hat{\lambda}] + \mathrm{Im}^2[\det\hat{\lambda}]}, \tag{14}$$

$$f_2(k_x,k_z) = \frac{\mathrm{Re}[\lambda_{zy}]\mathrm{Re}[\det\hat{\lambda}] + \mathrm{Im}[\lambda_{zy}]\mathrm{Im}[\det\hat{\lambda}]}{\mathrm{Re}^2[\det\hat{\lambda}] + \mathrm{Im}^2[\det\hat{\lambda}]}, \tag{15}$$

$$f_3(k_x,k_z) = \frac{(\lambda_{xx}\mathrm{Re}[\lambda_{zz}] - k_x^2 k_z^2)\mathrm{Re}[\det\hat{\lambda}] + \lambda_{xx}\mathrm{Im}[\lambda_{zz}]\mathrm{Im}[\det\hat{\lambda})}{\mathrm{Re}^2[\det\hat{\lambda}] + \mathrm{Im}^2[\det\hat{\lambda}]}, \tag{16}$$

$$f_4(k_x,k_z) = \frac{\lambda_{xx}\mathrm{Im}[\lambda_{zz}]\mathrm{Re}[det\hat{\lambda}] - (\lambda_{xx}\mathrm{Re}[\lambda_{zz}] - k_x^2 k_z^2)\mathrm{Im}[\det\hat{\lambda}]}{\mathrm{Re}^2[\det\hat{\lambda}] + \mathrm{Im}^2[\det\hat{\lambda}]}. \tag{17}$$

We note that while integrating in the (k_x, k_z)–space one should take into account different asymptotic expressions for $\lambda_{ij}(\omega = k_x V_0, k_y = 0, k_z)$ in dependence on the position in the (k_x, k_z)–plane. We use a semi–analytical method to solve these integrals, thereby applying the concept of asymptotic expansion of the relevant functions. This method allows us to obtain analytic expressions for the spatial behaviour of the electric and magnetic fields components far from the source, yielding a power–law decay along the line of motion of the conductor and an oscillatory behavior along the z–direction. Figure 1 shows the obtained solution for the x–component of the magnetic field $B_x(x, z)$ and the total value of the electric field $E^2(x, z) = E_x^2 + E_y^2 + Ez^2$. We note that these plots only correspond to the numerical value of the integrals (7)–(12) as a function of the spatial coordinates in cm. Taking into account that we take the source current I_0 equal to the current in the Io flux tube, the constant factors are $A_1 = 120.9$ statvolt/cm for the electric field and $A_2 = 5.1 \cdot 10^5$ gauss for the magnetic field.

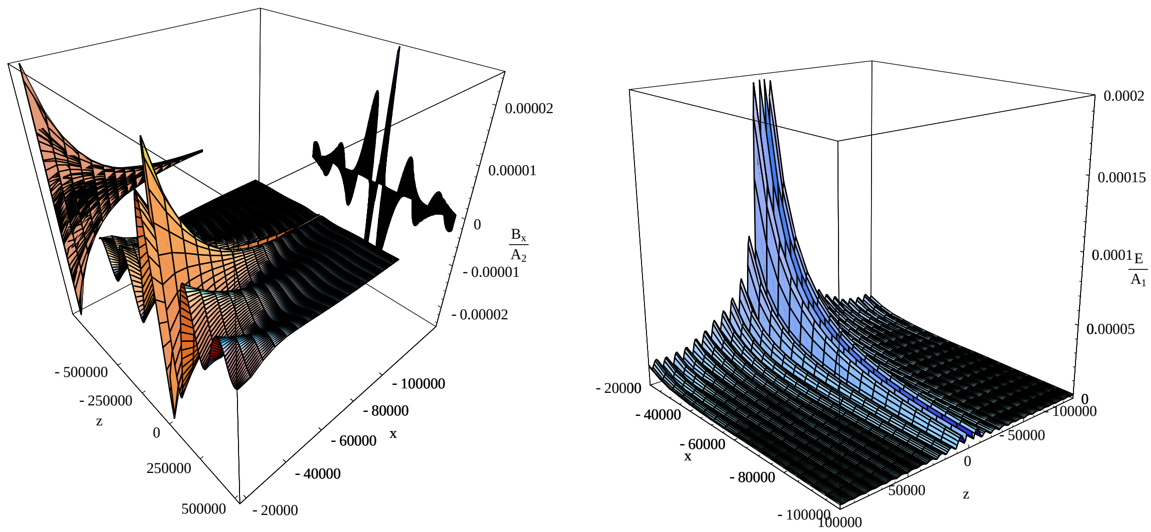

Figure 1: Plot of the x–component of the magnetic field (left) and the total amount of the electric field (right). Along the z–direction, i.e., the direction of the external magnetic field, we obtain a decaying oscillatory behaviour with a gradual damping.

The figures show a power law decay of the fields along the direction of the current source motion, i.e., along the x-direction. The electromagnetic environment just behind the source shows a stretched structure co-moving with the body.

3 Energy dissipation

In the previous section we obtained solutions for the electric and magnetic fields structure showing a stretched configuration. These fields decay in space due to a collisionless energy dissipation. This process is analogous to the anomalous skin–effect in a collisionless isotropic plasma. The important point is that this is a pure kinetic effect and it is impossible to detect it within the frame of MHD. The external current source's energy dissipated via this collisionless mechanism is equal to the work produced by the electric field $\mathbf{E}$ applied to the current source , $\mathbf{j}_s$, taken with another sign. Thus, in our case we have the following expression for the energy dissipation rate calculated per unit length along the y–axis

$$P_{unit} = \frac{16\pi I_0^2 V_0}{c^2(2\pi)^2} \int\limits_0^\infty \int\limits_0^\infty k_x \left(f_4(k_x,k_z)\right)_{k_y=0;\ \omega=V_0 k_x}\ dk_x dk_z. \tag{18}$$

The behaviour of this energy dissipation rate as a function of the current source velocity is shown in Figure 2.

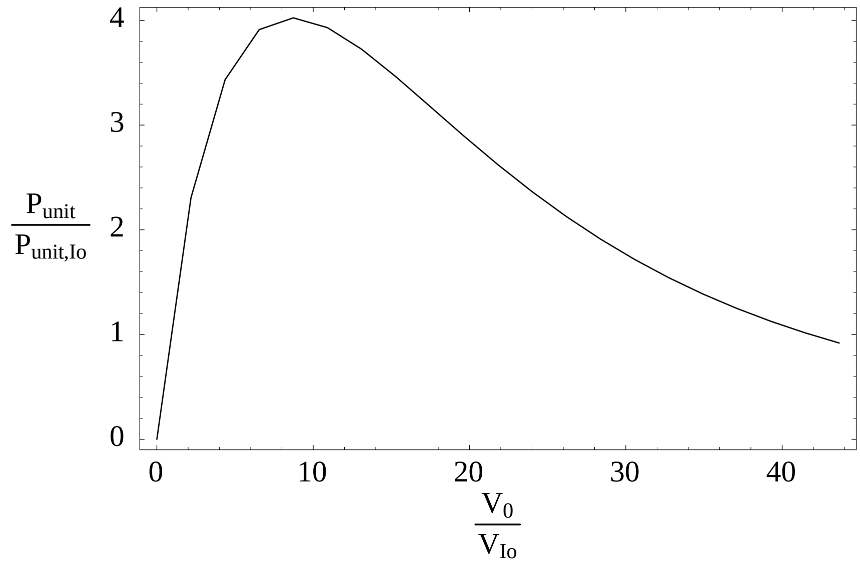

Figure 2: Energy dissipation rate as a function of the current source velocity. This rate is normalized to the energy dissipation rate when the conductor moves with the Io's velocity ($P_{unit,Io} = 3.8 \cdot 10^6$ erg/s), whereas the velocity is normalized to the velocity of Io ($V_{Io} = 5.2 \cdot 10^6$ cm/s).

In our 2D problem we deal with the infinite in y–direction current sources, then the total energy dissipated is infinite, too. Since, production of such an infinite current source requires an infinite energy source. All these difficulties are just a result of a theoretically idealized infinite current source structure considered here, which in fact can't be created in reality. At the same time, formula (18) well demonstrates the role and importance of the collisionless energy dissipation effects in energetics of a moving external current source and can be used in a qualitative analysis.

4 Conclusion

We consider the Io–Jupiter interaction as one of possible applications of the general plasma kinetics problem of a moving conducting body in a collisionless plasma. This model allows to develop methods for the interpretation of some aspects of the Jovian observational data, in particular, for some features of the Jovian decametric radio emission and measurements of the fields. The specific circumstances in the Jovian system oblige us to consider particular case of the model with a slowly moving conducting body, taking into account the effects of plasma particles thermal motion ($V_e \gg V_0 \gg V_i$). In this very case the low–frequency inductive electromagnetic fields will be effectively generated in the vicinity of the conductor. To emphasize the importance of these low-frequency inductive electromagnetic fields, which form some kind of a local magnetosphere around the moving conductor, and influence its energetic and radiative features, is the main goal of the present paper. We have shown that taking into account the particles thermal motion causes additional mechanism of energy dissipation with regard to the radiative energy loss mechanism. In spite of the non–self–consistent formulation of the problem of a given external current moving in a magnetized plasma, the results obtained here, can serve for testing the self–consistent solutions in the limiting cases. Besides, these elementary cases are also of interest for a spacecraft or antenna motion in the magnetized plasma of the ionosphere and magnetosphere.

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